Influence of Fluctuations on Electron Beam Diagnostics

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Theme

AN unresolved question regarding the accuracy of electron beam diagnostics in facilities characterized by unsteady test conditions has been the degree to which such measurements are influenced by fluctuations in flow properties. In most cases, photoelectric or photographic techniques are used to record time averaged spectral intensities which, if fluctuations are present, do not characterize the true time average of the property being measured. This synoptic investigates the influence fluctuations in rotational temperature, vibrational temperature, and number density may have on electron-beam measurements of nitrogen rotational temperature reduced with the line-slope technique.

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The analysis assumes the excitation-emission scheme formulated by Muntz.[1] Hence, perturbing effects caused by secondary electron excitation or preferential collisional quenching are ignored. In addition, although significant anomalies can result from an unsteady electron beam current, a constant current is assumed because, physically, current fluctuations will have the same effect as nitrogen number density fluctuations, i.e., both will produce variations in the number of fluorescing N_2^+ molecules.

With these assumptions the intensity of a rotational line reduces to a function of the nitrogen number density (N), rotational temperature (T_R), and vibrational temperature (T_V). In an unsteady flow their instantaneous values can be expressed as

$$T_R = \bar{T}_R \pm T_R' \tag{1a}$$

$$T_V = \bar{T}_V \pm T_V' \tag{1b}$$

$$N = \bar{N} \pm N' \tag{1c}$$

where the barred quantities are average values and the primed quantities are the fluctuating components.

Following Weeks,[2] the instantaneous intensity of a rotational line can be approximated by a Taylor series truncated to second order and expanded about $\bar{N}$, $\bar{T}_R$, $\bar{T}_V$. By time averaging the expansion, first-order terms drop out, leaving

$$\bar{I}_K = I_K(\bar{N}, \bar{T}_R, \bar{T}_V) + \tfrac{1}{2}\Big[\Big(\frac{\delta^2 I_K}{\delta T_V^2} \Big) \langle T_V'^2 \rangle + \Big(\frac{\delta^2 I_K}{\delta T_R^2} \Big) \times \langle T_R'^2 \rangle + \Big(\frac{\delta^2 I_K}{\delta N^2} \Big) \langle N'^2 \rangle + 2\Big(\frac{\delta^2 I_K}{\delta T_V \delta T_R} \Big) \langle T_V' T_R' \rangle + 2\Big(\frac{\delta^2 I_K}{\delta T_V \delta N} \Big) \langle T_V' N' \rangle + 2\Big(\frac{\delta^2 I_K}{\delta T_R \delta N} \Big) \langle T_R' N' \rangle \Big] \tag{2}$$

where the symbol $\langle\ \rangle$ also denotes the time average.

Details concerning electron beam nitrogen rotational temperature measurements can be found in Ref. 1. Ordinarily, the R-branch of the (0,0) band of the First Negative System of N_2^+ is used. In abbreviated form, the intensity of the Kth line in this band can be expressed as

$$I_K = X_K N \sum_{V_0} q(0, V_0) F_1(K, T_R, V_0) F_2(T_V, V_0) \tag{3}$$

where X is constant for a particular line, $q(0, V_0)$ denotes a Franck-Condon factor and F_1 and F_2 are functions of the rotational and vibrational temperatures, respectively.

Differentiating and substituting Eq. (3) into Eq. (2), a complicated expression for the average intensity of the Kth rotational line results. It can be simplified by normalizing the fluctuating quantities T_R', T_V', N' by their respective average values

$$\epsilon_1 = T_V' / \bar{T}_V \tag{4a}$$

$$\epsilon_2 = T_R' / \bar{T}_R \tag{4b}$$

$$\epsilon_3 = N' / \bar{N} \tag{4c}$$

The resulting equation for the average line intensity becomes

$$\bar{I}_K = X_K \bar{N} \{ a_1 + a_2 \bar{T}_V^2 \langle \epsilon_1^2 \rangle + a_3 \bar{T}_R^2 \langle \epsilon_2^2 \rangle + a_4 \bar{T}_V \bar{T}_R \langle \epsilon_2 \epsilon_1 \rangle + a_5 \bar{T}_V \langle \epsilon_1 \epsilon_3 \rangle + a_6 \bar{T}_R \langle \epsilon_2 \epsilon_3 \rangle \} \tag{5}$$

where the a_i are functions of F_i, F_2 and their respective derivatives.

When using the line-slope technique, in order to facilitate comparison between different sets of data, the line intensities are usually normalized with respect to one line in the band. Here, ratios have been formed with the fifth rotational line and have the form

$$\Big(\frac{\bar{I}_K}{\bar{I}_5} \Big) = \Omega \, \frac{1 + \dfrac{a_2}{a_1} \bar{T}_V^2 \langle \epsilon_1^2 \rangle + \dfrac{a_3}{a_1} \bar{T}_R^2 \langle \epsilon_2^2 \rangle}{1 + \dfrac{b_2}{b_1} \bar{T}_V^2 \langle \epsilon_1^2 \rangle \pm \dfrac{b_3}{b_1} \bar{T}_R^2 \langle \epsilon_2^2 \rangle} \left[\frac{+ \dfrac{a_4}{a_1} \bar{T}_V \bar{T}_R \langle \epsilon_1 \epsilon_2 \rangle + \dfrac{a_5}{a_1} \bar{T}_V \langle \epsilon_1 \epsilon_3 \rangle + \dfrac{a_6}{a_1} \bar{T}_R \langle \epsilon_2 \epsilon_3 \rangle}{+ \dfrac{b_4}{b_1} \bar{T}_V \bar{T}_R \langle \epsilon_1 \epsilon_2 \rangle + \dfrac{b_5}{b_1} \bar{T}_V \langle \epsilon_1 \epsilon_3 \rangle + \dfrac{b_6}{b_1} \bar{T}_R \langle \epsilon_2 \epsilon_3 \rangle} \right] \tag{6}$$

where Ω is the intensity ratio which would result for steady test conditions and the b_i correspond to the a_i of Eq. (5), but are evaluated for $K = 5$. Thus the bracketed expression is a correction factor which encompasses the effects of fluctuations on the measurement. Since Eq. (3) is a linear function of N, $\bar{N}$ cancels upon ratioing, and Eq. (6) does not explicitly depend on the number density. An implicit dependence arises only through the definition of ϵ_3.

When reducing electron-beam data, the ratioing procedure corresponds to dividing Eq. (3) by a constant. If the rotational states are in equilibrium, a plot of the left-hand side

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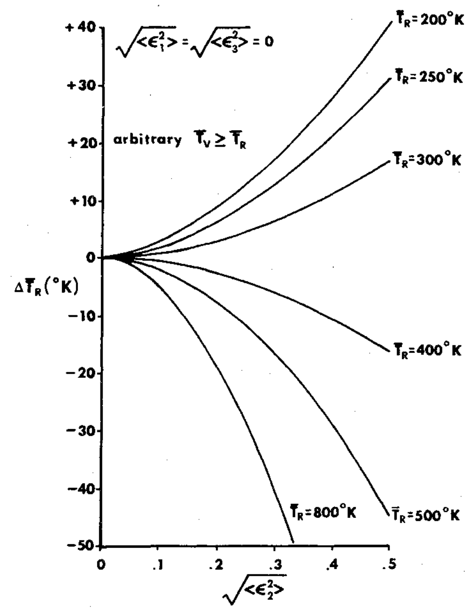

Fig. 1 Error due to rotational temperature fluctuations.

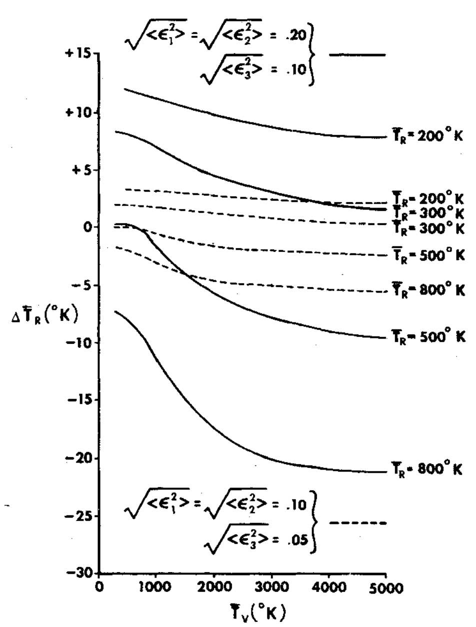

Fig. 2 Resulting error in perfectly correlated cases.

of this expression against $K(K+1)$ results in a straight line whose slope is directly related to $\bar{T}_R$. However, since the expression is a function of $\bar{T}_R$, the procedure is not straight forward, and $\bar{T}_R$ can only be obtained through iteration. Numerically $\bar{T}_R$ was determined with a least mean squares procedure which was repeated until successive values of the temperature agreed to within 0.1%.

The effects of unsteadiness have been investigated for two statistically limiting cases.[3] In the first case, the fluctuating components have been assumed to be uncorrelated, (i.e., $\langle\epsilon_i\epsilon_j\rangle = 0$, with $i \neq j$), and to separate the effects due to T_R' from those caused by T_V', one or the other has been assumed to be dominant. In the second case, positive correlation has been assumed, i.e.,

$$\langle\epsilon_i\epsilon_j\rangle = (\langle\epsilon_i^2\rangle)^{1/2}(\langle\epsilon_j^2\rangle)^{1/2}$$

The accuracy of the Taylor series calculations were arbitrarily set to be within 11%. For the perfectly correlated

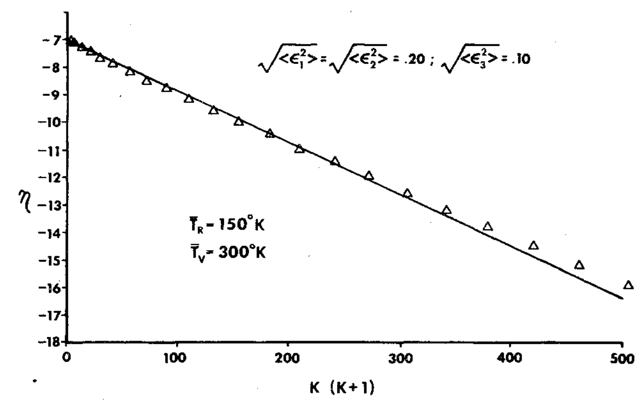

Fig. 3 Nonlinearity caused by fluctuations. η is a nondimensional function proportional to ln $[\bar{I}_k / \bar{I}_5]$.

cases this allowed maximum values of 0.2 for both $(\langle\epsilon_1^2\rangle)^{1/2}$ and $(\langle\epsilon_2^2\rangle)^{1/2}$, and 0.1 for $(\langle\epsilon_3^2\rangle)^{1/2}$. For the uncorrelated cases this permitted values up to 0.5 for $(\langle\epsilon_1^2\rangle)^{1/2}$ or $(\langle\epsilon_2^2\rangle)^{1/2}$ and $(\langle\epsilon_3^2\rangle)^{1/2} = 0.05$.

Results: When the only fluctuations present are T_V', the effects on the measurements are found to be experimentally negligible. Similarly when only N' are present, Eq. (6) reduces to Ω, i.e., the result is independent of all fluctuations and electron-beam measurements give the true average rotational temperature. This, however, is not the case when rotational fluctuations are present. Figure 1 shows the resulting effects on the measured value of $\bar{T}_R$. If the true $\bar{T}_R$ is below 350K, the measured $\bar{T}_R$ is too high and the relative error between the measured and true value increases with decreasing values of true $\bar{T}_R$. Above 400K, the measured values are lower than the true values. Thus, when the fluctuations are uncorrelated, only rotational temperature fluctuations have a significant effect on the measurements.

Figure 2 shows the error in $\bar{T}_R$ which occurs in the case of perfect positive correlation characterized by fluctuations in all 3 variables simultaneously. The measured $\bar{T}_R$ is seen to decrease with increasing $\bar{T}_R$. The strong dependence of ΔT_R on $\bar{T}_V$ is caused by the $\langle\epsilon_i\epsilon_j\rangle$ terms in Eq. (6) which, although small in themselves, have a pronounced effect when $\bar{T}_V$ is large. The larger fluctuations are seen to produce the worst results. In addition, although uncorrelated number density fluctuations have no effect on the measurements, they do produce a noticeable effect when they are correlated with both rotational and vibrational temperature fluctuations. These results, together with the role played by individual ϵ_i, are discussed in detail in the original paper.

Observed nonlinearity in line slope plots: Spectral scans of electron beam induced radiation often produce line slope plots showing a nonlinear behavior for large values of K.[4] A similar nonlinear trend can be generated by fluctuations. Figure 3 is a typical example, calculated for $\bar{T}_R = 150$ K with $(\langle\epsilon_1^2\rangle)^{1/2} = (\langle\epsilon_2^2\rangle)^{1/2} = 0.2$; $(\langle\epsilon_3^2\rangle)^{1/2} = 0.1$, which shows higher K value points diverging from a straight line.

References

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